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Citation: [AIP Conference Proceedings](#) **1580**, 514 (2014); doi: 10.1063/1.4864601

View online: <http://dx.doi.org/10.1063/1.4864601>

View Table of Contents: <http://scitation.aip.org/content/aip/proceeding/aipcp/1580?ver=pdfcov>

Published by the [AIP Publishing](#)

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# Nonlinear 2-D Effects in the Control of Magnetic Islands by ECCD

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**Abstract.** *The stabilization of tearing magnetic islands by means of localized current driven by electron cyclotron waves, requires optimizing the efficiency of the injected helical current. The problem is conventionally addressed using 0-D model of the (generalized) Rutherford equation to find the dependence in terms of the island width, wave beam width and deposition scale length, as well as phase tracking requirements. The use of a 2-D reconnection model shows that both the early time response of a tearing unstable system to ECCD and important nonlinear processes lead to irreversible modifications on the 2-D configuration, where “phase” and “width” of an island cease to be observable and controllable state variables. In particular the occurrence of a phase instability and of multiple axis and current sheets, may be a serious impediment for feedback control schemes .*

**Keywords:** Tearing mode, magnetic reconnection, ECCD, control, phase instability

**PACS:** 52.35.Vd, 52.30.Cv, 52.35.Py, 52.35.Hr, 52.55.Wq

## INTRODUCTION

An attractive method of control of the tearing instabilities in a tokamak is based on the localized injection, within a magnetic island, of a current driven by the absorption of electron cyclotron waves [1-3]. A number of experimental results, obtained on several devices have demonstrated successful quenching of the perturbations, but several questions remain open on the physical process of control of the instabilities and on the consequent best strategies to be applied. Key problems in achieving robust control consist in finding the conditions of maximal efficiency of the helical ECCD current in terms of the island width, wave beam width and deposition scale length, and in determining the phase tracking strategy. At present practical control systems are being designed on the basis of the 0-D model of the (generalized) Rutherford equation, which describes the time evolution of the nominal width of the island. The 0-D Rutherford equation models [4] predict very stringent requirements of the focusing of the EC wave beam on the magnetic island to achieve its full suppression and rely on keeping track of both phase and amplitude of the perturbation. In such approach, basic topological aspects of the problem cannot be properly accounted for. In the present work new information is provided on the 2D and time dependent effects of the ECCD action on the magnetic islands, that is very important to formulate applicable control strategies, and also to help with the interpretation of experimental observations.

## RMHD MODEL OF ECCD CONTROLLED RECONNECTION

Here the full response of a magnetic island to an ECCD control, is studied numerically starting from the spontaneous magnetic reconnection process occurring in a 2-D RMHD “Harris pinch” equilibrium model. The analysis during the linear, the algebraic and in the saturated stage of the reconnection process, is performed in the frame of the Reduced Resistive Magneto-Hydro-Dynamics (RRMHD) description [5] in terms of the scalar helical flux function  $\psi$  and the parallel vorticity  $U$ . In dimensionless notation the total magnetic field is expressed as  $\mathbf{B} = B_{0z}\mathbf{e}_z + \nabla\psi \times \mathbf{e}_z$ , the current density is  $\mathbf{J} = \mathbf{e}_z \cdot \nabla \times \mathbf{B}_\perp = -\nabla^2\psi$  and the velocity and vorticity fields are related by  $U = \mathbf{e}_z \cdot \nabla \times \mathbf{v}_\perp = \nabla^2\varphi$  where  $\varphi$  is the stream function; the subscript 0 labels the equilibrium quantities. All

lengths are scaled to the macroscopic equilibrium B field scale length  $L$ , while the time is normalized on the Alfvén time  $\tau_A = L/|B_{y,0}|$ . The governing nonlinear equations are then cast in the form:

$$\begin{aligned} \frac{\partial \psi}{\partial t} + \mathbf{v}_\perp \cdot \nabla \psi &= -\eta(J - J_0 + J_{ec}) \\ \frac{\partial U}{\partial t} + \mathbf{v}_\perp \cdot \nabla U - \mathbf{B}_\perp \cdot \nabla J &= 0 \end{aligned} \quad (1)$$

The control current, driven by an EC wave beam, is peaked on the (elliptic) O-point of an island, whose geometric width is related to the amplitude of the perturbed magnetic field:

$$J_{ec}(x, y, t) = J_m(t) e^{-\psi(x, y, t) - \psi_0(t) / \delta^2}. \quad (2)$$

This current density is uniformly distributed on  $\psi = \text{const.}$  contours around the flux label of the O-point, with an effective deposition width  $\delta$ , in flux units. The effect of the control current  $J_{ec}$  is to restore the ideal frozen flux condition in Eq. (1), by reducing the current density perturbation  $J - J_0$ . It is expected to counteract the unstable reconnection process and eventually suppress the magnetic island. The amplitude  $J_m$  is proportional to the difference between the maximum and the minimum current density values on the rational surface  $x = x_s$  at  $t = t_1$ ,  $J_m = a(J_{\max}(t_1) - J_{\min}(t_1))$ . The system of Eqs. (1) is integrated numerically by the code used in [6]. The calculations are carried out in a large  $\Delta' = 2(1/k_y - k_y)$  regime, with  $\eta \sim 5 \cdot 10^{-4}$  and a current perturbation  $O(10^{-4})$ :  $\delta J(x, y) = \hat{j}(x) \cos k_y y$ .

## SIMULATION OF CONTROLLED RECONNECTION REGIMES

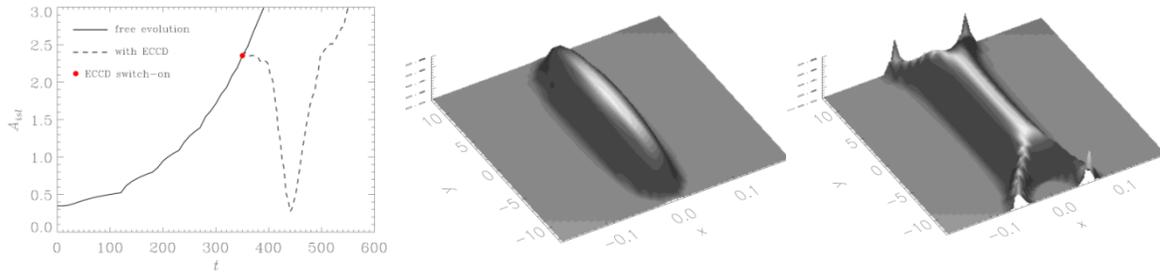
The historical evolution of the reconnection process is described in terms of the free growth of the effective area  $A_{is}(t)$  enclosed by the separatrix of the magnetic island. The basic response of the tearing mode perturbation to the  $J_{ec}$  injection is studied with amplitude of  $J_m$  in the intervals  $1 \leq J_m / (J_X - J_O)_{t_1} \leq 10$  and  $0.1 \leq b \equiv \delta / (\psi_X - \psi_O)_{t_1} \leq 1$  where X and O label the values at the X and at the O point of the magnetic island, respectively.

After a short transient, four stages can be identified. In the first (I), for  $250 < t < 450$  the single helicity magnetic perturbation exhibits an exponential growth in time [7]. Then at  $t = 500$  the nonlinear regime starts, lasting till  $t = 900$ , during which the magnetic island growth is algebraic in time. It should be noted that in this test case the algebraic growth stage (II) does *not* lead to the saturation regime. Indeed a second exponential growth starts around  $t = 900$  and it lasts till  $t = 1100$ , then saturation follows (IV) [8]. The results of the application of  $J_{ec}$  are shown in the three frames of Fig. 1, for a fixed value of the amplitude and width of the ECCD. The island is suppressed around  $t \sim 430$ , but its area bounces back with a growth rate equal to the previous rate of quench. In the small island approximation  $w^2 = -8(\psi_X - \psi_O) / J_{0,x=0}$  our equilibrium and current drive choices, lead to the following equation for  $w^2$ :

$$\frac{\partial w^2}{\partial t} = 8\eta \left( J_X - J_O \pm J_m \left( e^{-w^4/8\delta^2} - 1 \right) \right) \quad (3)$$

Eq. (3) shows that the growth of the magnetic island depends on the balance of the total current ( $J + J_{ec}$ ) between the X- and O- points of the island which depends nonlinearly on the injected current,  $J_{ec}$ . In other words, the effect of the applied current meant to restore stability in the linear stage (so called “early” control action) may lead on the contrary to another unstable state. This is a state bifurcation known as flip instability [9,10] because the value of the reconnected flux at the resonant surface changes sign, which is equivalent to a shift of  $L_y/2$  of the elliptic O-point of the tearing perturbation. This behavior, already studied in the context of classical reconnection driven by boundary condition perturbations reappears here, with universal characteristics, in the frame of the so-called non-inductive current drive effects where it has been always ignored. After the first flip the magnetic island grows monotonically. This behavior can be understood recalling that at the flip the island is suppressed and, consequently, the ECCD contribution to Eq. (3) becomes negligible. However the equilibrium configuration is still unstable, and the magnetic

island start growing again, driven by the positive sign of the term  $J_X - J_O$  on the rhs. of Eq. (3) which is dominant, just after the flip.

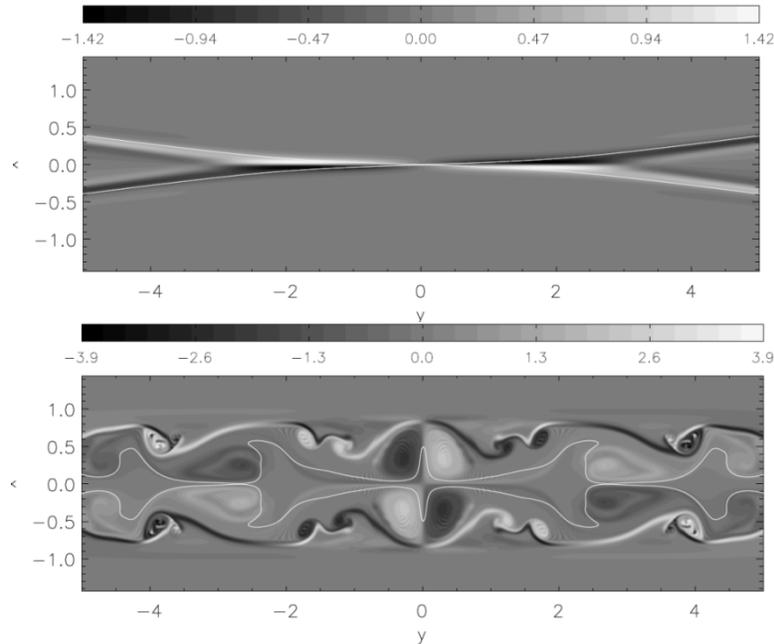


**FIGURE 1.** LEFT - Area of the magnetic island vs. time for the current drive with amplitude  $a=10$  and width  $b=0.1$ . CENTER and RIGHT - ECCD distribution before ( $t = 415$ ) and after ( $t = 470$ ) the flip.

The injected current at the X-point drives new unstable modes that lead to a strong modification of the magnetic island and to a *total loss* of the control. In the early nonlinear regime (II) the control current is turned on at  $t_1 = 800$ . Unlike the results obtained in the linear stage of the reconnection instability under ECCD control, in all the cases considered starting from a macroscopic magnetic island, in the nonlinear stage, a complete suppression of the magnetic island was never achieved. Low values of the peak amplitude of the current, i.e.  $a \leq 2$ , have proved rather ineffective in quenching the main  $m$  component of the magnetic flux function perturbation. The magnetic island area reduces almost monotonically on much longer time scales than those observed in the linear stage [11]. A faster island contraction is observed when  $a \geq 3$ . However, the nonlinear growth of higher order harmonics in the magnetic perturbation prevents the island suppression. This is due to the onset of a secondary Kelvin-Helmholtz (K-H) instability [12] which affects the strongly sheared plasma flows appearing at the initial stages of the control process. In every case the K-H instability affects both the velocity and vorticity fields and responsible for the deformation of the magnetic island separatrix, as shown in Fig. 2. The magnetic surfaces  $\psi = const.$  are advected by the plasma flow towards the resonant surface where they are forced to reconnect and secondary island chains are formed. In such complex topology the island “phase” and “width” are no longer observable and controllable variables that can be used in a control action. The evolution of the magnetic island area always exhibits a flip, and the growth of the island after the flip depends strongly on the amplitude of the ECCD. As shown recently [13-16], the ECCD into a macroscopic island leads to a new equilibrium configuration. Two thin current sheets parallel to the  $y$  axis are inductively formed, placed symmetrically on both sides of the resonant surface  $x_s = 0$ . In the saturation stage (IV) the ECCD has been turned on at time  $t = 1500$  of the free system evolution. Just as for the ECCD control in the algebraic stage (II), also in this case the magnetic island *never shrinks* to zero. A new equilibrium is reached on the time scale the free system takes to reach saturation, with the response of two inductive current sheets on each side of the singular surface and a macroscopic irreversible deformation of the magnetic topology.

## CONCLUSIONS

The analysis of finite 2-D nonlinear effects shows that a successful control action can be achieved with broader ECCD current injection, less demanding from the control point of view, and providing an averaged effect over the magnetic island, closer to the conditions of the Rutherford model, with a more gradual reduction of the magnetic island width. However a broader ECCD beam has a lower control efficiency, since a relevant fraction of the injected current falls outside the separatrix, even for small magnetic island reduction. For finite width islands, the effects occurring on different space scales, strongly change the perturbation spectrum, with the appearance of secondary harmonics and the irreversible modification of the original magnetic equilibrium, which reduce the control action, expected specifically to decrease the primary, most unstable harmonic. In addition, phase tracking feedback schemes are challenged by the flip instability and bang-bang (optimal control) strategies may be more suitable.



**FIGURE 2.** Blowup around the point  $(0,0)$  of the field  $U$  before (top panel) and after (bottom panel) the ECCD injection.

## ACKNOWLEDGEMENTS

This work was partly supported by the Euratom Communities under the contract of Association between EURATOM/ENEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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